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Integrating the Outliers

Two Lessons from the St. Petersburg Paradox

The risk-reducing formulas behind portfolio theory rely on a number of demanding and ultimately unfounded premises. First, they suggest that price changes are statistically independent from one another . . . The second assumption is that price changes are distributed in a pattern that conforms to a standard bell curve.

Do financial data neatly conform to such assumptions? Of course, they never do.

Benoit B. Mandelbrot
A Multifractal Walk down Wall Street [1]

The very fact that the Petersburg Problem has not yielded a unique and generally acceptable solution to more than 200 years of attack by some of the world's great intellects suggests, indeed, that the growth-stock problem offers no hope of a satisfactory solution.

David Durand
Growth Stocks and the Petersburg Paradox [2]

con · sil'i · ence, *n.* [con- + salire to leap]
interlocking explanations of cause and effect between disciplines

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Bernoulli's Challenge

Competent investors take great pride in their ability to place an appropriate value on a financial claim. This ability is the core of investing: Markets are just vehicles to trade cash for future claims, and vice versa.

O.K. Here's a cash flow stream for you to value: Say the house flips a fair coin. If it lands on heads, you receive $2 and the game ends. If it lands on tails, the house flips again. If the second flip lands on heads, you get $4; if it lands on tails, the game continues. For each successive round, the payoff for landing on heads doubles (i.e., $2, $4, $8, $16, etc.) and you progress to the next round until you land heads. How much would you pay to play this game?

Daniel Bernoulli, one of a family of distinguished mathematicians, first presented this problem to the Imperial Academy of Sciences in 1738.[3] Bernoulli's game, known as the St. Petersburg Paradox, challenges classical theory, which says that a player should be willing to pay the game's expected value to participate. The expected value of this game is infinite. Each round has a payoff of $1 (probability of $1/2^n$ and a payoff of $\$2^n$, or ½ x $2, ¼ x $4, ⅛ x $8, etc.) So,

Expected value = 1 + 1 +1 + 1 . . . = ∞

Naturally, very few people would be willing to pay even $20 to play the game. Bernoulli tried to explain the paradox with the marginal utility of money. He argued that the amount you'd be willing to pay is a function of your resources—the greater your resources, the more you'd be willing to pay. Still, Bernoulli's explanation is not altogether satisfactory. The St. Petersburg Paradox has kept philosophers, mathematicians, and economists thinking for over two-and-a-half centuries.[4]

Philosophical discourse aside, the St. Petersburg Paradox illuminates two very concrete ideas for investors. The first is that the distribution of stock market returns does *not* follow the pattern that standard finance theory assumes. This deviation from theory is important for risk management, market efficiency, and individual stock selection.

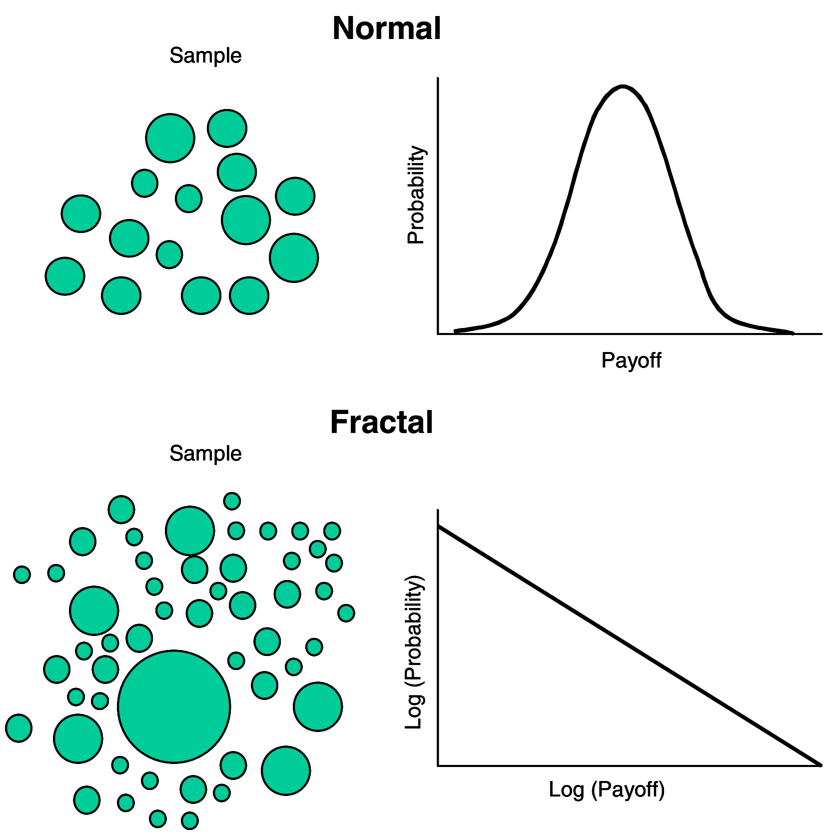
The second idea relates to valuing growth stocks. What do you pay today for a business with a low probability of an extraordinarily high payoff? This question is more pressing than ever in a world with violent value migrations and increasing returns.

What's Normal?

Asset price distributions are of great practical significance for portfolio managers. Standard finance theory assumes that asset price changes follow a normal distribution—the well-known bell curve. That this assumption is roughly accurate most of the time allows analysts to use very robust probability statistics. For example, for a sample that follows a normal distribution, you can identify the population average and characterize the likelihood of variance from that average.

However, much of nature—including the man-made stock market—is not normal.[5] Many natural systems have two defining characteristics: an ever-larger number of smaller pieces and similar-looking pieces across the different size scales. For example, a tree has a large trunk and a number of ever-smaller branches, and the small branches resemble the big branches. These systems are fractal. Unlike a normal distribution, no average value adequately characterizes a fractal system. Exhibit 1 contrasts normal and fractal systems visually and shows the probability functions that represent the data. Fractal systems follow a power law.[6]

Exhibit 1: Probability Density Functions for Normal and Fractal Systems

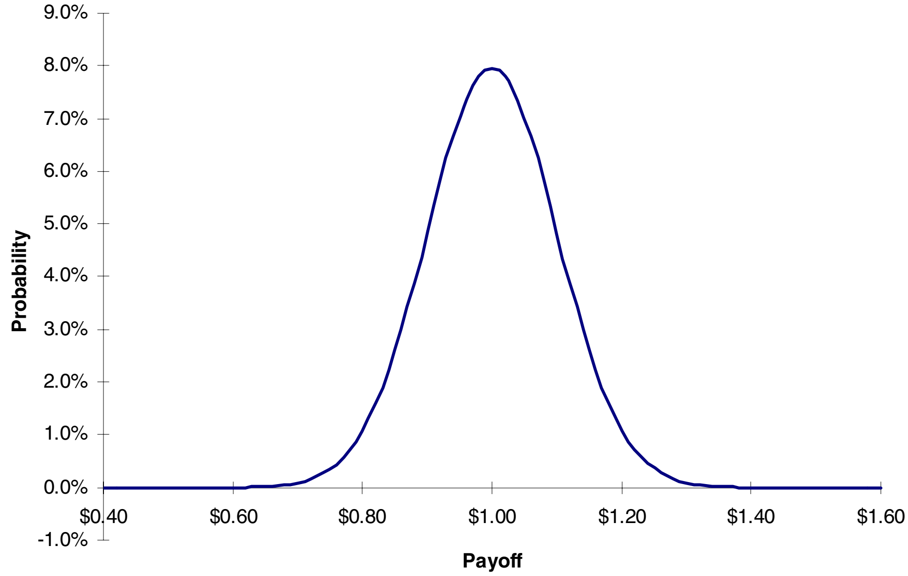


Source: Larry S. Liebovitch and Daniela Scheurle, "Two Lessons from Fractals and Chaos," *Complexity*, Vol. 5, 4, 2000.

Using the statistics of normal distributions to characterize a fractal system like financial markets is potentially very hazardous. Yet theoreticians and practitioners do it daily.[7] The distinction between the two systems boils down to probabilities and payoffs. Fractal systems have few, very large observations that fall outside the normal distribution. The classic example is the crash of 1987. The probability (assuming a normal distribution) of the market's 20%-plus plunge in one day was so infinitesimally low it was practically zero. And still the losses were a staggering $2 trillion-plus.

A comparison of a normal coin toss game and the St. Petersburg game illustrates the point. Assume that you toss a coin and receive $2 if it lands heads and nothing if it lands tails. The expected value of the game is $1, the amount you would be willing to pay to play the game in a fair casino. We simulated 1 million rounds of 100 tosses each, and plotted the payoffs in Exhibit 2. Just as you would expect, we got a well-defined normal distribution.[8]

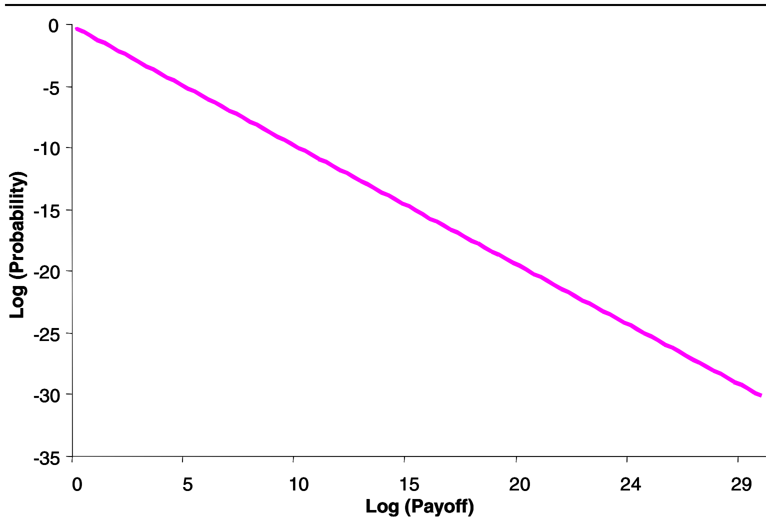
Exhibit 2: Standard Coin Toss Game



Source: CSFB analysis.

We then simulated the St. Petersburg game 1 million times, and plotted that distribution (see Exhibit 3). While the underlying process is stochastic, the outcome is a power law. For example, half the time the game only pays $2, and three-quarters of the time it pays $4 or less. However, a run of 30 provides a $1.1 billion payoff, but is only a 1-in-1.1 billion probability. Lots of small events and a few very large events characterize a fractal system. Further, the average winnings per game is unstable with the St. Petersburg game, so no average accurately describes the game's long-term outcome.

Exhibit 3: Fractal Coin Toss Game



Source: CSFB analysis.

Are stock market returns fractal? Benoit Mandelbrot shows that by lengthening or shortening the horizontal axis of a price series—effectively speeding up or slowing down time—prices series are indeed fractal. Not only are rare large changes interspersed with lots of smaller one, the price changes look similar at various scales (e.g., daily, weekly, and monthly returns). Mandelbrot calls financial time series multifractal, adding the prefix "multi" to capture the time adjustment.

In an important and fascinating book, *Why Stock Markets Crash*, geophysicist Didier Sornette argues that stock market distributions comprise two different populations, the body (which you can model with standard theory) and the tail (which relies on completely different mechanisms). Sornette's analysis of market drawdowns convincingly dismisses the assumption that stock returns are independent, a key pillar of classical finance theory. His work provides fresh and thorough evidence of finance theory's shortcomings.[9]

St. Petersburg and Growth Stock Investing

The St. Petersburg Paradox also provides insight for growth stock valuation.[10] What should you be willing to pay for a very small probability that a company can grow its cash flows by a very significant amount forever?[11]

David Durand took up this question in his classic 1957 article, "Growth Stocks and the Petersburg Paradox."[12] He encourages a good deal of caution, emphasizing reversion-to-the-mean thinking and modeling. But if anything, the challenge to value the low probability of significant value is even more pressing today than it was when Durand took on the challenge 45 years ago.

Consider, for example, that of the nearly 2,000 technology initial public offerings since 1980, only 5% account for over 100% of the $2-trillion-plus in wealth creation.[13] And even within this small wealth-generating group, only a handful delivered the bulk of the huge payoffs. Given the winner-take-most characteristics of many growth markets, there's little reason to anticipate a more normal wealth-creation distribution in the future.

In addition, the data show that the distribution of economic return on investment is wider in corporate America today than it was in the past.[14] So the spoils awaiting the wealth creators, given their outsized returns, are greater than ever before. Like the St. Petersburg game, the majority of the payoffs from future deals are likely to be modest, but some will be huge. What's the expected value? What should you be willing to pay to play?

Integrating the Outliers

The St. Petersburg Paradox may be centuries old, but its lessons are as fresh as ever. One of the major challenges in investing is how to capture (or avoid) low-probability, high-impact events. Unfortunately, standard finance theory has little to say about the subject.

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[1] Benoit B. Mandelbrot, "A Multifractal Walk down Wall Street," *Scientific American*, February 1999, 70-73.
[2] David Durand, "Growth Stocks and the Petersburg Paradox," *Journal of Finance*, 12, September 1957, 348-363.
[3] Daniel Bernoulli, "Exposition of a New Theory on the Measurement of Risk," *Econometrica*, 22, January 1954, 23-36. Originally published in 1738. Daniel's cousin, Nicolaus, initially proposed the game.
[4] See http://plato.stanford.edu/entries/paradox-stpetersburg/.
[5] Much of this section relies on Larry S. Liebovitch and Daniela Scheurle, "Two Lessons from Fractals and Chaos," *Complexity*, Vol. 5, 4, 2000, 34-43. See http://www.ccs.fau.edu/~liebovitch/complexity-20.html.
[6] Michael J. Mauboussin and Kristen Bartholdson, "More Power to You: Power Laws and What They Mean for Investors," *The Consilient Observer*, September 24, 2002.
[7] If you assume that you flipped a coin nonstop 16 hours a day (estimating 8 hours of sleep), and if each coin flip takes three seconds, it would take 14.3 years to complete 100 million coin tosses.
[8] Mandelbrot. Also, Benoit B. Mandelbrot, *Fractals and Scaling in Finance* (New York: Springer Verlag, 1997).
[9] Didier Sornette, *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton: Princeton University Press, 2003). See http://www.ess.ucla.edu/faculty/sornette/.
[10] See another classic article: Peter L. Bernstein, "Growth Companies vs. Growth Stocks, " *Harvard Business Review*, September-October 1956.
[11] Peter L. Bernstein, *Against the Gods: The Remarkable Story of Risk* (New York: John Wiley & Sons, 1996), 107-109.
[12] Durand.
[13] Stephen R. Waite, *Quantum Investing* (New York: Texere, 2003), 129.
[14] Michael J. Mauboussin, Bob Hiler, and Patrick J. McCarthy, "The (Fat) Tail that Wags the Dog," *Credit Suisse First Boston Equity Research*, February 4, 1999.

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